Bayesian Image Reconstruction From Retinal Cone Signals

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Collaborator:
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Ideal observer analysis for simple stimulus

Spehar et al., 2015; Geisler 1989; Banks et al., 1987
Our approach

Natural Images

Ideal Observer

Contrast sensitivity

Spehar et al., 2015; Geisler 1989; Banks et al., 1987
Our approach

Spehar et al., 2015; Geisler 1989; Banks et al., 1987
Bayesian image reconstruction from retinal cone signals

Part I

stimulus $s$

Part II

cone excitation $e$

Bayesian estimation

$\arg\max p(e \mid s)p(s)$

Part III

Cottaris et al., 2019
Bayesian image reconstruction from retinal cone signals

Part I

stimulus \( s \)

\[ p(e \mid s) \]

cone excitation \( e \)

Part II

Bayesian estimation

\[ \text{argmax} \quad p(e \mid s)p(s) \]

Part III

ISETBio - github.com/isetbio/isetbio
Model of early vision

Cottaris et al., 2019
Model of early vision

Cottaris et al., 2019
Model of early vision

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Model of early vision

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Model of early vision
Model of early vision

Cottaris et al., 2019
Model of early vision

Cottaris et al., 2019
Bayesian image reconstruction from retinal cone signals

Part I

stimulus $s$

cone excitation $e$

likelihood

$p(e \mid s)$

$p(e \mid s) = \prod_{k=1}^{K} \text{Poisson}(e_k \mid \lambda_k = [\text{ISETBio}(s)]_k)$
Bayesian image reconstruction from retinal cone signals

Part I

stimulus $s$

cone excitation $e$

$\mathbb{P}(e \mid s) = \prod_{k=1}^{K} \text{Poisson}(e_k \mid \lambda_k = [\text{ISETBio}(s)]_k)$

Part II

Bayesian estimation

$\text{argmax} \quad p(e \mid s)p(s)$

Part III
Prior distribution of natural images

\[ \approx -0.1 \times \begin{array}{c} \text{image} \\ \end{array} + 0.5 \times \begin{array}{c} \text{image} \\ \end{array} + \ldots + 0.2 \times \begin{array}{c} \text{image} \\ \end{array} \]

Olshausen & Field, 1996
Prior distribution of natural images

\[ p(\beta) \]

\[ \approx -0.1 \times \beta_1 + 0.5 \times \beta_2 + \ldots + 0.2 \times \beta_n \]
Prior distribution of natural images

\[ p(\beta) \]
Maximum a posteriori estimation

cone excitation $e$ \hspace{10cm} estimate $\hat{s}$

\[ \arg\max \quad \log p(e | \hat{s}) + \log p(\hat{s}) \]

- $p(e | \hat{s}) = \prod_{k=1}^{K} \text{Poisson}(e_k | \lambda_k = [\text{ISETBio}(\hat{s})]_k)$

- $p(\hat{s}) = \prod_{n=1}^{N} \text{Laplace}(\beta_n)$
Maximum a posteriori estimation

cone excitation $e$ \hspace{2cm} estimate $\hat{s}$

Bayesian estimation

$$\arg\max \log p(e | \hat{s}) + \gamma \log p(\hat{s})$$

- $p(e | \hat{s}) = \prod_{k=1}^{K} \text{Poisson}(e_k | \lambda_k = [\text{ISETBio}(\hat{s})]_k)$

- $p(\hat{s}) = \prod_{n=1}^{N} \text{Laplace}(\beta_n)$
Bayesian image reconstruction from retinal cone signals

\[ L(s, \hat{s}) \]

stimulus \( s \)  
128*128*3  

cone excitation \( e \)  
1-deg, ~10000 cones

\[
p(e \mid s) = \prod_{k=1}^{K} \text{Poisson}(e_k \mid \lambda_k = [\text{ISETBio}(s)]_k) \]

Bayesian estimation  
\[
\arg\max \quad p(e \mid s)p(s) 
\]

\[
p(s) = \prod_{n=1}^{N} \text{Laplace}(\beta_n) \]

Part III
Allocation of cone type

![Graph showing allocation of cone type with RMSE values and percentage of cone.]
Allocation of cone type

Garrigan et al., 2010
Effect of image statistics

Increase spatial correlation

Increase chromatic correlation

- Percentage of L Cone in Mosaic
- Average RMSE of Reconstruction

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Increase in Spatial Correlation

Increase in Chromatic Correlation
Effect of image statistics

Increase spatial correlation

Increase chromatic correlation

Average RMSE

%L Cone
Dichromacy

RMSE vs. %L Cone

RMSE vs. %S Cone

S(L+M): 0:1 9:1
L:M: 0:1 1:4 2:3 3:2 4:1
L:0

S: 0 1 9:1

S(L+M): 5 20 40 60 80 95

%L Cone: 0 10 20 40 60 80 90 100

%S Cone: 0 10 20 40 60 80 90 100
Reconstruction for dichromacy

Original Image

Protanopia

Deuteranopia

Tritanopia
Reconstruction for dichromacy

Original Image

Protanopia
Reconstruction for dichromacy

Original Image

Protanopia

Deuteranopia
Reconstruction for dichromacy

Original Image

Protanopia

Deuteranopia

Tritanopia
Reconstruction for dichromacy

Original Image

Protanopia

Bayesian Image Reconstruction

Summary

- Image computable model of early vision
- Ideal observer analysis with Bayesian image reconstruction

\[ p(e \mid s) = \prod_{k=1}^{K} \text{Poisson}(e_k \mid \lambda_k = [\text{ISETBio}(s)]_k) \]

\[ p(s) = \prod_{n=1}^{N} \text{Laplace}(\beta_n) \]

- Optimal design of retinal mosaic
- Visualization of dichromacy