**Fig. S1** The experimental apparatus and patterns used to investigate how honeybees use visual information to control speed when landing. The experimental apparatus: side (A) and front (B) elevations. C) The patterns used to investigate if speed control in landing is affected by the spatio-temporal frequency of the visual information.

**Text S1. Derivation of the optic flow profile that is generated when a honeybee approaches a stationary disc**

Consider a honeybee approaching a disc along its axis at a distance of $z$ with a forward motion of $V$ perpendicular to the plane of the disc, as shown in Fig. S2. What is the apparent value of angular velocity of any point on the disc that will be perceived by the honeybee’s eye? If we take a point on the disc that is at a distance $r$ from its centre and at a viewing direction of $\theta$ from the position of the honeybee then

$$\tan \theta = \frac{r}{z} \quad (S1)$$

Therefore the value of $r$ can be written

$$r = z \cdot \tan \theta \quad (S2)$$
Fig. S2 Variables used in the landing model. (A) Illustration of the model variables when a honeybee approaches a vertical surface. $V$ represents the axial velocity (measured as the change in distance over time) of the bee approaching the disc on a horizontal trajectory, $z$ is the perpendicular (axial) instantaneous distance of the bee from the disc, $r$ is the radial distance from the approach point to a stationary point on the disc and $\theta$ is the viewing angle (the angle with respect to the flight direction). Light grey arrows represent the pattern of radial expansion that a bee would experience as it approaches a vertical surface (B) Illustration of the model variables when a bee approaches a vertical disc displaying an exponential spiral pattern. $R$ is the radial distance to a point of contrast on the arm of the spiral pattern.

When approaching a stationary disc, the value of $z$ will vary with time as the distance between the honeybee and the disc decreases but the value of $r$ at any given point on the disc will remain constant. Thus, to calculate the change in $\theta$ over time under this condition, we differentiate (S1) with respect to time whilst keeping $r$ constant:

$$\frac{1}{\cos^2 \theta} \cdot \frac{d\theta}{dt} = r \cdot \left( -\frac{1}{z^2} \right) \cdot \frac{dz}{dt}$$  \hspace{1cm} (S3)
Given that

\[ V = - \frac{dz}{dt} \]  

(S4)

we can insert (S4) into (S3) to obtain

\[ \frac{d\theta}{dt} = \frac{Vr}{z^2} \cdot \cos^2 \theta \]  

(S5)

Inserting the expression for \( r \) from (S2), (S5) can be expressed as

\[ \frac{d\theta}{dt} = \frac{V}{z} \cdot \tan \theta \cdot \cos^2 \theta \]  

(S6)

Simplifying (S6), we obtain

\[ \frac{d\theta}{dt} = \frac{V}{2z} \cdot \sin 2\theta \]  

(S7)

where \( \frac{d\theta}{dt} \) is the perceived angular velocity of a point at radius \( r \) as the honeybee approaches the disc.

Text S2. Derivation of the properties of a spiral pattern that will generate the same optic flow profile whether it is stationary or rotating

To change the magnitude of the image motion experienced on the approach to a vertical surface, without changing the profile of optic flow experienced when the spiral is stationary, it is necessary to design a spiral that has specific geometric properties. To calculate these properties, it is first important to determine how the distance between the centre of the spiral and a point of high contrast on the spiral \( R \) (i.e. a point along the edge of one of the spiral arms whose distance is not constant but changes with when the spiral rotates, see Figure 1), changes over time when the spiral is rotated while the target distance \( (y) \) remains constant. To calculate the change in \( \theta \) over time under this condition, we need to differentiate (1) with respect to time whilst keeping \( z \) constant:

\[ \frac{1}{\cos^2 \theta} \cdot \frac{d\theta}{dt} = \left( \frac{1}{z} \right) \cdot \frac{dR}{dt} \]  

(S8)

Rearranging (S8) with respect to \( \frac{d\theta}{dt} \) we obtain

\[ \frac{d\theta}{dt} = \left( \frac{1}{z} \right) \cdot \frac{dR}{dt} \cdot \cos^2 \theta \]  

(S9)

To find the properties of the spiral that are necessary for it to produce the same optic flow profile when it is rotating as when it is stationary and being approached, it is necessary that the value of \( \frac{d\theta}{dt} \) given in (S5) is proportional to the value of \( \frac{d\theta}{dt} \) that is given in (S9).
Thus,
\[
\left(\frac{1}{z}\right) \cdot \frac{dR}{dt} \cdot \cos^2 \theta = \alpha \cdot \frac{VR}{z^2} \cdot \cos^2 \theta
\]  
(S10)
where \( \alpha \) represents the constant of proportionality.

Rewriting (S10):
\[
\frac{dR}{R} = \alpha \cdot \frac{v}{z} \cdot dt
\]  
(S11)

Integrating (10) with respect to time, we obtain the following expression for \( R \):
\[
R = A \cdot e^{\frac{av}{z} \cdot t}
\]  
(S12)
where \( A \) represents a constant of proportionality. If the spiral rotates at a constant speed, the angular velocity of \( \omega \) is constant and, for a constant value of \( \omega \), the angle of rotation \( \psi \) can therefore be described as
\[
\psi = \omega \cdot t
\]  
(S13)

So by inserting (S13) into (S12) we obtain
\[
R = A \cdot e^{\frac{av \cdot \psi}{\omega z}}
\]  
(S14)
or
\[
R = A \cdot e^{B \cdot \psi}
\]  
(S15)
where \( B \) is defined as
\[
B = \frac{av}{\omega z}
\]  
(S16)
where \( z, V \) and \( \omega \) are constant.

This result demonstrates that, for a rotating spiral to generate the same optic flow profile as it does when it is stationary, the distance of point \( R \) from the centre of the disc must increase exponentially with the angle from the centre. This means that the radius of an arm of the spiral must increase exponentially, with a pitch value of \( B \). The spiral patterns that are used in the experiments described in this study had a pitch of \( B = 0.3 \).
Text S3. Derivation of the perceived angular velocity of point R when a stationary honeybee views the rotating spiral

From (S9), the angular velocity of the rotating spiral stimulus when \( z \) is constant is

\[
\frac{d\theta}{dt} = \left( \frac{1}{x} \right) \cdot \frac{dR}{dt} \cdot \cos^2 \theta \quad (S17)
\]

To obtain the value of \( R \) as it changes with time, we differentiate (S15) with respect to time:

\[
\frac{dR}{dt} = A \cdot B \cdot e^{B \cdot \psi} \cdot \frac{d\psi}{dt} = A \cdot B \cdot e^{B \cdot \psi} \cdot \omega = B \cdot R \cdot \omega = B \cdot \omega \cdot z \cdot \tan \theta \quad (S18)
\]

Substituting (S18) into (S17):

\[
\frac{d\theta}{dt} = \frac{B \cdot \omega \cdot z \cdot \tan \theta}{z} \cdot \cos^2 \theta \quad (S19)
\]

Thus,

\[
\frac{d\theta}{dt} = \frac{B \cdot \omega}{z} \cdot \sin 2\theta \quad (S20)
\]

From (S20) we see that the apparent angular rate of expansion of the spiral (\( \frac{d\theta}{dt} \)) will be positive if \( \omega \) is positive (the spiral rotates to produce apparent expansion) and negative if the spiral rotates in the opposite direction. At any constant viewing distance (\( z \)), the magnitude of the optic flow profile, and hence the value of angular velocity (\( \frac{d\theta}{dt} \)) of \( R \), at any constant viewing angle (\( \theta \)), will increase with positive values of \( \omega \) and decrease for negative values of \( \omega \). Under this condition, the angular velocity of \( R \) is zero when the spiral is stationary.

Text S4. Derivation of the angular velocity at a viewing direction \( \theta \) of point R when a honeybee approaches a rotating spiral pattern

The angular velocity perceived by a honeybee at any given viewing direction as it approaches the rotating spiral can be described by adding the component of angular velocity produced when the static spiral is approached (as obtained in (S7)) with the component of angular velocity generated when the spiral is rotating but the viewer’s position (i.e. \( z \)) is constant (as obtained in (S20)):

\[
\frac{d\theta}{dt} = \left( \frac{V}{2z} + \frac{B \omega}{z} \right) \cdot \sin 2\theta \quad (S21)
\]

Therefore, if the distance of the bee from the spiral (\( z \)), the instantaneous speed of the honeybee (\( V \)), the pitch of the spiral (\( B \)) and the angular rotation (\( \omega \)) of the spiral are known, then the above expression can be used to calculate the angular velocity (\( \frac{d\theta}{dt} \)) of the image of the spiral at the honeybee’s eye at any viewing angle (\( \theta \)). For a given viewing angle \( \theta \), the quantity (\( \sin 2\theta \)) is a constant (say, \( K \)). Thus we may write
\[
\frac{d\theta}{dt} = K \left( \frac{V}{2z} + \frac{B\omega}{2} \right)
\]

When the spiral is stationary \((\omega = 0)\), we have \(\frac{d\theta}{dt} = \frac{KV}{2z}\). Therefore, if the honeybees are holding the value of \(\frac{d\theta}{dt}\) constant for some constant viewing angle \(\theta\) while approaching a stationary spiral, the ratio \(\frac{V}{z}\) should remain constant during the landing process. This means that a plot of \(V\) versus \(z\) (approach speed versus distance) should be a straight line with a slope of \(\frac{2d\theta}{Kdt}\) passing through the origin. If the spiral is rotating at an angular velocity of \(\omega\), then the plot of \(V\) versus \(z\) will still be a straight line, but with a slope \(\frac{2d\theta}{Kdt} - B\cdot\omega\). This slope will be lower than the static case when the spiral expands (positive \(\omega\)) and higher than the static case when the spiral contracts (negative \(\omega\)). The change in the slope of the \(V\)-\(z\) relationship will depend upon the pitch of the spiral \((B)\) and its rotational speed \((\omega)\).

If the bees are regulating their approach to the rotating spiral by holding some value of the perceived angular velocity constant then:

\[
\frac{V_{\text{stat}}}{2z} = \left( \frac{V_{\text{rot}}}{2z} + \frac{B\cdot\omega}{2} \right) \quad \text{(S22)}
\]

or

\[
\frac{V_{\text{rot}}}{z} = \frac{V_{\text{stat}}}{z} - B\omega \quad \text{(S23)}
\]

The bees are holding some value of angular velocity constant if the slope of the relationship between \(V\) and \(z\) when the pattern is rotating, \(V_{\text{rot}}\), is equal to the value of the slope when the pattern is stationary, \(V_{\text{stat}}\) minus the value of \(B\omega\).

**Text S5. Derivation of the optic flow profile generated when landing on a plane of arbitrary orientation**

Fig. S3 shows a bee flying at a velocity \(V\) along the \(z\) direction of a Cartesian co-ordinate system (origin at O), during an approach to land on an arbitrarily oriented plane (shown in blue). The normal to the plane is depicted by the vector ON. The fronto-parallel \((x-y)\) plane is shown in grey.
Fig. S3 Calculation of the optic flow produced by a point P on a plane inclined at an arbitrary angle to the direction of an approaching bee.

Consider a fixed point P on the inclined plane. OM and OB are the projections of the line OP on the x-y plane and the z axis, respectively. We denote the lengths of AO, AB, PB, and AP by z, r, a, and s, respectively, and the angles POA and PAO by \( \theta \) and \( \alpha \), respectively. \( \alpha \) represents the direction of the point P in the bee’s visual field. \( \phi \) denotes the angle between OM and the y axis, measured in the x-y plane. If the bee approaches a fronto-parallel plane, \( \theta \) will be 90 deg for any point P on the plane. But for an arbitrarily oriented plane (as shown in the figure), \( \theta \) will vary as a function of \( \phi \), and this function will depend upon the orientation of the plane. We denote this function by \( \theta(\phi) \).

We have

\[
\tan \alpha = \frac{a}{r} \tag{S24}
\]

The angular velocity of the point P in the bee’s eye is \( \frac{d\alpha}{dt} \). To determine this angular velocity, we differentiate (S24) with respect to time to obtain

\[
\sec^2 \alpha \frac{d\alpha}{dt} = -\frac{a}{r^2} \frac{dr}{dt} \tag{S25}
\]

Noting that \( \frac{dr}{dt} = -V \), we can write (S25) as

\[
\sec^2 \alpha \frac{d\alpha}{dt} = -\frac{a}{r^2} (-V) = V \frac{a}{r^2} \tag{S26}
\]

Noting that \( \cos \alpha = \frac{r}{\sqrt{a^2+r^2}} \), we can write (S26) as

\[
\frac{d\alpha}{dt} = V \frac{a}{a^2+r^2} = \frac{V}{\sqrt{a^2+r^2}} \frac{a}{\sqrt{a^2+r^2}} = \frac{V}{s} \sin \alpha \tag{S27}
\]
where \( s = \sqrt{a^2 + r^2} \).

Noting that \( s = \frac{r}{\cos \alpha} \), we can write (S27) as

\[
\frac{da}{dt} = \frac{V}{r} \cdot \sin \alpha \cdot \cos \alpha
\]  
(S28)

Now, from the geometry of the triangles APB and OPB, we have

\[ a = r \cdot \tan \alpha = (z - r) \tan \theta \]  
(S29)

which yields: \( r = \frac{z \tan \theta}{\tan \alpha + \tan \theta} \)  
(S30)

Inserting (S30) into (S28), we obtain

\[
\frac{da}{dt} = \frac{V}{z} \frac{\tan \alpha + \tan \theta}{\tan \theta} \sin \alpha \cos \alpha = \frac{V}{z} \left(1 + \tan \alpha \tan \theta \right) \sin \alpha \cos \alpha
\]  
(S31)

i.e.

\[
\frac{da}{dt} = \frac{V}{z} \sin \alpha \cos \alpha + \frac{V}{z} \frac{\sin^2 \alpha}{\tan \theta}
\]  
(S32)

Denoting by \( \omega \) the angular velocity in the eye of the point \( P \), we may write

\[
\omega = \frac{V}{z} \sin \alpha \cos \alpha + \frac{V}{z} \frac{\sin^2 \alpha}{\tan \theta}
\]  
(S33)

The first term on the right-hand side of (S33) represents the angular velocity of the point \( P \) if it lies in a fronto-parallel plane at a distance \( z \) from the bee. The second term represents the correction to this value if the point lies in a plane that is not fronto-parallel – this term is zero when the plane is fronto-parallel (in which case \( \theta = 90^\circ \), making \( \tan \theta = \infty \)). For a plane of arbitrary orientation, \( \theta \) is a function of \( \phi \).

Equn (S33) can be rewritten as

\[
\omega = \frac{V}{z} \left[ \sin \alpha \cos \alpha + \frac{\sin^2 \alpha}{\tan \theta} \right]
\]  
(S34)

This equation indicates that the angular velocity in the eye (\( \omega \)) along any viewing direction \( \alpha \), in a plane of arbitrary orientation, depends upon the ratio of the velocity of approach (\( V \)) to the distance to the target (\( z \)). Therefore, if \( \omega \) along any viewing direction is held constant, the velocity of approach will decrease steadily as the target is approached, in accordance with the landing control law that we propose in this study.

Let us next consider the case when the visual system monitors the magnitude of the optic flow not just in a single direction, but also over a cone of visual space, of semi-angle \( \alpha_0 \), in the frontal visual field. The magnitude OF of the optic flow integrated over this cone would be given by:
\[ \text{OF} = \int_0^{2\pi} d\phi \int_0^{\alpha_0} \omega \, d\alpha \quad (S35) \]

Inserting the expression for \( \omega \) from (S34) into (S35), we obtain:

\[
\text{OF} = \int_0^{2\pi} d\phi \int_0^{\alpha_0} \frac{V}{z} \left[ \sin \alpha \cos \alpha + \frac{\sin^2 \alpha}{\tan \theta} \right] \, d\alpha
\]

\[
= \frac{V}{z} \int_0^{2\pi} d\phi \int_0^{\alpha_0} \alpha \sin \alpha \cos \alpha \, d\alpha + \frac{V}{z} \int_0^{\alpha_0} \alpha \sin^2 \alpha \, d\alpha \int_0^{2\pi} \frac{d\phi}{\tan \theta(\theta)} \quad (S36)
\]

which can be expressed as:

\[
\text{OF} = \frac{V}{z} \left[ 2\pi A(\alpha_0) + C B(\alpha_0) \right] \quad (S37)
\]

where \( A(\alpha_0) = \int_0^{\alpha_0} \alpha \sin \alpha \cos \alpha \, d\alpha \), \quad (S38)

\( B(\alpha_0) = \int_0^{\alpha_0} \alpha \sin^2 \alpha \, d\alpha \) \quad (S39)

and \( C = \int_0^{2\pi} \frac{d\phi}{\tan \theta(\theta)} \quad (S40) \)

We note that:

\( A(\alpha_0) = \int_0^{\alpha_0} \frac{\alpha \sin 2\alpha}{2} \, d\alpha \quad (S41) \)

which can be integrated (by parts) to obtain

\[
A(\alpha_0) = \frac{1}{8} \left[ \sin 2\alpha_0 - 2\alpha_0 \cos 2\alpha_0 \right] \quad (S42)
\]

Similarly, \( B(\alpha_0) \) can be integrated to obtain

\[
B(\alpha_0) = \frac{1}{4} \left[ \alpha_0^2 - \alpha_0 \sin 2\alpha_0 + \sin^2 \alpha_0 \right] \quad (S43)
\]

We may therefore rewrite (S36) as

\[
\text{OF} = \frac{V}{z} \frac{\pi}{4} (\sin 2\alpha_0 - 2\alpha_0 \cos 2\alpha_0) + \frac{C}{4} (\alpha_0^2 - \alpha_0 \sin 2\alpha_0 + \sin^2 \alpha_0) \quad (S44)
\]

where \( C = \int_0^{2\pi} \frac{d\phi}{\tan \theta(\theta)} \)

Equation (S44) indicates that the magnitude of the optic flow integrated over a cone of visual space (of arbitrary size) depends upon the ratio of the velocity of approach \( (V) \) to the distance to the target \( (z) \). Therefore, if this integrated optic flow is held constant, the velocity of approach will decrease steadily as the target is approached, again in accordance with the landing control law that we propose in this study.
This result is true for a plane inclined at any orientation relative to the direction of approach. We also note that the result is not restricted to planes. It holds for any surface that can be generated by pivoting a line about the target point – such as a cone, a ridge, or a flower-like surface (Figure S4). Each of these surfaces is described by a particular $\theta(\phi)$ function, and therefore by a particular value of $C$. In the case of a fronto-parallel plane, $C = 0$. The second term in the expression of the integrated optic flow in Equation (S44) represents a correction that is applied to all surfaces other than a fronto-parallel plane. The strategy that we have described in this study can therefore be applied to landing on a large variety of surfaces.

Fig. S4 Examples of surfaces for which the landing strategy can be applied: (A) an arbitrarily inclined plane; (B) a cone (C) a ridge; (D) a flower-like surface. All of these surfaces can be generated by pivoting a line about the target point, as described in the text.

Text S6. Derivation of the model of Taudot-guidance for landing

6.1 Variation of approach velocity with target distance

We denote the approach speed by $V$, and the distance to the target by $z$. The instantaneous time to contact, $Tau$, is then given by

$$Tau = \frac{z}{V} \quad (S45)$$
If we denote the rate of change of Tau (Taudo) by \( p \), we can write

\[
p = \frac{d\text{Tau}}{dt} = \frac{d}{dt} \left( \frac{z}{V} \right) \tag{S46}
\]

Integrating (S46) with respect to time, we can write

\[
z = V(pt + U) \tag{S47}
\]

where \( U \) is a constant of integration.

Noting that \( V = -\frac{dz}{dt} \), and inserting this into Equation (S47), we obtain

\[
\frac{dz}{z} = -\frac{dt}{pt + U} \tag{S48}
\]

which can be integrated to obtain

\[
z = \frac{W}{(pt + U)^p} \tag{S49}
\]

or \( z^p(pt + U) = W^p \) \( \tag{S50} \)

Substituting for \((pt + U)\) from (S47) and denoting \( W^p \) by another constant \( E \), we can write

\[
V = \frac{z^{p+1}}{E} \tag{S51}
\]

Equation (S51) specifies the relationship between the target distance \((z)\) and the approach velocity \((V)\) for any prescribed value of Taudo \((p)\). The constant \( E \) depends upon the initial conditions of \( z \) and \( V \). Setting \( V_{\text{initial}} = 1018 \text{ mm/sec} \) and \( z_{\text{initial}} = 300 \text{ mm} \), we can calculate \( E \) separately for each value of \( p \):

For \( p = -0.5 \), \( E = 0.017 \)
For \( p = 0.0 \), \( E = 0.295 \)
For \( p = +0.5 \), \( E = 5.104 \)

The relationship described by Equation (S51) is plotted in Figure S5A (for a stationary spiral) for \( p = -0.5, 0.0 \) and \(+0.5\) using the corresponding values of \( E \).
**Fig. S5** Prediction of holding $\text{Taudot}$ constant when landing. (A) The predicted approach speed versus target distance (for a stationary spiral) for holding $\text{Taudot} \ (p)$ constant at -0.5 (red line), 0.0 (black line) and +0.5 (blue line). We see that 0.0 is the only value of $p$ that produces a linear relationship between $V$ and $z$. Our experimental results (Figures 2-4) indicate a linear relationship. (B) Predicted maximum image angular velocity versus target distance (for a stationary spiral) when holding $\text{Taudot} \ (= p)$ constant at at -0.5 (red line), 0.0 (black line) and +0.5 (blue line).

### 6.2 Variation of image angular velocity with target distance

For a stationary spiral, Equation (S7) specifies the angular velocity $\omega$ of the image in a viewing direction $\theta$ for an approach velocity $V$:

$$\frac{d\theta}{dt} = \frac{V}{2z} \sin 2\theta \quad \text{(S52)}$$

Therefore the maximum angular velocity (which occurs at $\theta = 45$ deg) is given by

$$\omega_{45} = \frac{V}{2z} \quad \text{(S53)}$$
Substituting for $V$ from (S51), we obtain

$$\omega_{45} = \frac{2p}{2E}$$  

(S54)

Equation (S54) is plotted in Figure S5 (for a stationary spiral) for $p = -0.5, 0.0$ and $+0.5$, using corresponding values of $E$.

We see that 0.0 is the only value of $p$ that makes the maximum image angular velocity independent of target distance, as observed in the experiments. $p = 0.5$ and $p = +0.5$ produce very different behaviours.

Thus, in honeybees, control of landing is a special case of the general rule of holding $Taudot$ constant. In the case of the bee $Taudot$ is held constant at a value of zero, which means that $Tau$ is maintained at a fixed value.

**Text S7. Comparison with behaviour of Drosophila landing on a cylinder**

Our results indicate that, when honeybees approach a flat vertical surface, the speed of approach is approximately proportional to the distance to the surface. van Breugel and Dickinson (6) filmed trajectories of *Drosophila* landing on a vertical cylinder. They found that approach speed was approximately proportional to the logarithm of the distance of the fly to the centre of the cylinder (their Figs. 8A, 9A). Here we examine whether our model, for bees landing on a flat vertical surface, also explains their findings.

We have shown above (Equation S7) that, for a bee approaching a vertical surface at a speed $V$ (see Fig. S2B), the perceived angular velocity in a direction $\theta$, when the bee is at a distance $z$ from the surface will be:

$$\frac{d\theta}{dt} = \frac{V}{2z} \cdot \sin 2\theta$$

We have shown that landing honeybees hold $(V/z)$ constant. Denoting this constant by $A$, we can rewrite the above relationship as

$$\frac{d\theta}{dt} = \frac{A}{2} \cdot \sin 2\theta$$  

(S55)

Thus, as a landing honeybee approaches the center of a vertical stripe the angular velocity of the edge as perceived by the eye would be specified by (S55), when the edge is at an angle $\theta$ with respect to the flight direction. ($\theta$ will increase as the stripe is approached, causing this angular velocity to change according to (S55)).

Now consider *Drosophila* approaching the center of a vertical cylinder of radius $r$ at a speed of $V'$, as shown in Fig. S6A. Here $\theta$ is the instantaneous direction of the boundary of the cylinder relative to the fly’s flight direction.
Fig. S6 Model of *Drosophila* approaching a cylinder. (A) Variables involved in the analysis of an approach to land on a cylinder. \( r \): cylinder radius; \( V' \): instantaneous approach speed; \( z \): instantaneous distance from center of cylinder; \( \theta \): instantaneous direction of cylinder boundary relative to flight direction. (B) Predicted variation of approach speed with distance from the center of the cylinder, plotted as a function of linear distance (blue) and as a function of the natural logarithm of the distance (red).

If the instantaneous distance to the center of the cylinder is \( z \), we have

\[
\sin \theta = \frac{r}{z} \quad (S56)
\]

Differentiating (S56) with respect to time and noting that \( \frac{dz}{dt} = -V' \), we obtain

\[
\cos \theta \frac{d\theta}{dt} = -\frac{r}{z^2} \frac{dz}{dt} = \left(\frac{V'}{z}\right) \frac{r}{z} \quad (S57)
\]

Noting that \( \frac{r}{z} = \sin \theta \), we may rewrite (S57) as

\[
\frac{d\theta}{dt} = \left(\frac{V'}{z}\right) \frac{\sin \theta}{\cos \theta} = \left(\frac{V'}{z}\right) \tan \theta \quad (S58)
\]
i.e. \( \frac{d\theta}{dt} = \left( \frac{V'}{z} \right) \tan \theta \) \hspace{1cm} (S59)

Thus, as a landing Drosophila approaches the center of a vertical cylinder the angular velocity of the boundary of the cylinder as perceived by the eye would be specified by (S59), when the edge is at an angle \( \theta \) with respect to the flight direction. \( \theta \) will increase as the stripe is approached, causing this angular velocity to change according to (S59).

If we assume that the same guidance strategy is used while approaching a vertical planar stripe or a vertical cylinder, then the rate of change of \( \theta \) (for any given value of \( \theta \)) should be the same in both cases. That is, we require \( \frac{d\theta}{dt} \) as specified by (S55) to be the same as \( \frac{d\theta}{dt} \) as specified by (S59). Equating the right-hand sides of these two expressions, we obtain

\[
\left( \frac{V'}{z} \right) \tan \theta = \frac{A}{2} \cdot \sin 2\theta
\]

which leads to

\[
V' = \frac{Az}{2} \cdot 2 \sin \theta \cos \theta \frac{\cos \theta}{\sin \theta} = Az \cos^2 \theta = Az \left( \frac{z^2 - r^2}{z^2} \right) = Az \left( 1 - \frac{r^2}{z^2} \right)
\]

Thus, while approaching a cylinder, the variation of the approach speed \( V' \) with the distance \( z \) to the center of the cylinder is predicted to be

\[
V' = Az \left( 1 - \frac{r^2}{z^2} \right)
\]

Fig. S6B shows the predicted relationship between \( V' \) and \( z \), assuming \( r = 1.0 \) distance units and \( A = 1.0 \) (units of \( s^{-1} \)). The actual values are not important for this qualitative discussion. We observe that this relationship is nonlinear (blue curve), unlike the situation for approach toward a stripe, and that it has a convex-upwards profile. On the other hand, the relationship between \( V' \) and \( \log(z) \) is approximately linear (red curve), at least for distances that are comparable to the radius of the cylinder. Thus, the predictions of our model are in qualitative agreement with the results reported by van Breugel and Dickinson (6).